Measure Preserving Flows for Ergodic Search in Convoluted Environments

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***Abstract*—Autonomous robotic search has important applications in robotics, such as the search for signs of life after a disaster. When *a priori* information is available, for example in the form of a distribution, a planner can use that distribution to guide the search. Ergodic search is one method that uses the information distribution to generate a trajectory that minimizes the ergodic metric, in that it encourages the robot to spend more time in regions with high information and proportionally less time in the remaining regions. Unfortunately, prior works in ergodic search do not perform well in complex environments with obstacles such as a building's interior or a maze. To address this, our work presents a modified ergodic metric using the Laplace-Beltrami eigenfunctions to capture map geometry and obstacle locations within the ergodic metric. Further, we introduce an approach to generate trajectories that minimize the ergodic metric while guaranteeing obstacle avoidance using measure-preserving vector fields. Finally, we leverage the divergence-free nature of these vector fields to generate collision-free trajectories for multiple agents. We demonstrate our approach via simulations with single and multi-agent systems on maps representing interior hallways and long corridors with non-uniform information distribution. In particular, we illustrate the generation of feasible trajectories in complex environments where prior methods fail.**
[1]

I. INTRODUCTION

Robotic exploration has multiple applications including search and rescue missions [1], autonomous data gathering/monitoring [2], and surveillance/patrolling [3]. Such exploration problems typically entail coverage path planning [4]–[6] where the robot determines a trajectory that visits every point in a given space. When *a priori* information is available, not every point needs to be visited, and it is beneficial to focus the robot's exploration on higher information regions. In this context, ergodic search [7] is a promising effort to aid the robots to spend more time in higher information regions and proportionally less in low information regions. However, virtually all prior efforts with ergodic search presume no or small obstacles in the search region [7]–[10]. In this work, we focus on generating feasible ergodic trajectories for uniform and non-uniform information maps in complex environments for multiple robots, as illustrated in orange, green, and purple in Fig. 1.

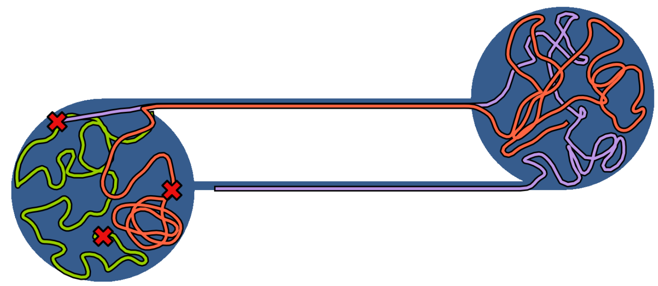

Fig. 1. An interior environment with two rooms connected by a narrow hallway. An ergodicity-minimizing trajectory generated using measure-preserving vector fields translates the three robots, starting from the red X's, to the other room through the narrow hallway for efficient information gathering.

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II. METHODS

We use the ergodic metric [7] used in many other ergodic search formulations to maximize coverage in a finite time horizon. However, rather than the standard Fourier basis functions for a square map $\phi_{ij} = \cos(k_i \pi x)\cos(k_j \pi y)$, we instead use the spatial harmonic basis functions using the eigenfunctions of the Laplace-Beltrami operator [11] (1).

$$\nabla^2 \phi_k = \lambda_k \phi_k \tag{1}$$

The kth spectral coefficient of the information distribution $p(x)$ is $\hat{\xi}_k = \langle p, \phi_k \rangle_M$ and the kth spectral coefficient of the agent trajectory is given as

$$\xi_k = \frac{1}{T}\sum_{t=1}^{T} \langle \phi_k, \delta_{x_t} \rangle_M = \frac{1}{T}\sum_{t=1}^{T} \phi_k(x_t) \tag{2}$$

The ergodic metric itself is the squared error between the information distribution and trajectory's spectral coefficients, weighted to place more importance on the low-frequency coefficients (3). The weighting formula $(1+\sqrt{\lambda_k})^{-2}$ is chosen to ensure consistent behavior on an obstacle-free square map when compared to prior literature.

$$\mathcal{E} = \sum_{k=0}^{\infty} \left(1+\sqrt{\lambda_k}\right)^{-2} \left(\xi_k - \hat{\xi}_k\right)^2 \tag{3}$$

A. *Measure-Preserving Flows*

To minimize the ergodic metric (3) on a finite time horizon, we use gradient-based optimization algorithms to find local minima. The challenge lies in finding feasible ergodic trajectories that do not intersect the non-convex obstacles. For this, we turn to measure-preserving flows (4).

$$\nabla \cdot (p(x)\vec{v}(x)) = 0 \tag{4}$$

As the solutions to (4) form a linear subspace, we can approximate that subspace to a certain extent using a finite basis of vector fields, all of which satisfy the measure-preserving criterion. Our paper focuses primarily on an application to a 2D setting, and one method for finding measure-preserving vector fields on 2D spaces is to use the 2D curl. First, we choose a set of basis functions $\{u_i(x)\}$ as the eigenfunctions of the Laplacian, subject to a Dirichlet boundary condition $u(x) = 0$ for all $x \in \partial X$.

$$\nabla^2 u_i = \lambda_i u_i \tag{5}$$

Then we can construct vector fields using the 2D curl, where x and y denote the two dimensions. We can see that the vector fields in equation (6) always satisfy (4), and on the boundary ∂X, v will always point parallel to the boundary.

$$v_i(x, y) = \left[-\frac{1}{p(x,y)} \frac{\partial u_i(x,y)}{\partial y}, \frac{1}{p(x,y)} \frac{\partial u_i(x,y)}{\partial x} \right] \tag{6}$$

B. Ergodic metric minimization

To minimize the ergodic metric (3) on a finite time horizon, we formulate the problem as a minimization problem over the state x_a^t for every agent at every time and vector field coefficients $u_i(t)$.

$$\begin{aligned} \min_{x_a^t, u_i(t)} \quad & \mathcal{E}(x_a^t) \\ \text{s.t.} \quad & \dot{x}_a^t = \sum_i u_i(t) v_i(x_a^t) \qquad \forall\, t \in [0, T] \\ & \|\dot{x}_a^t\| \le v_{\max} \qquad \forall\, t \in [0, T] \end{aligned} \tag{7}$$

Optimization is performed using gradient descent while respecting the constraints.

III. Results and Discussion

The trajectories computed for a chosen map are illustrated in Fig. 2, comparing our method (orrange) against competing methods. These results show that our method is able to traverse the long corridors and explore the full area. We observe that STOEC [12] is unable to cross from the left room to the right, as the corridors are too narrow. And while the control barrier approach [13] is able to cross over, it often gets stuck with parts of the trajectory in forbidden regions. In summary, our method not only explores both rooms successfully in all the test cases but also minimizes the ergodic metric, as evidenced by the lower ergodicity values.

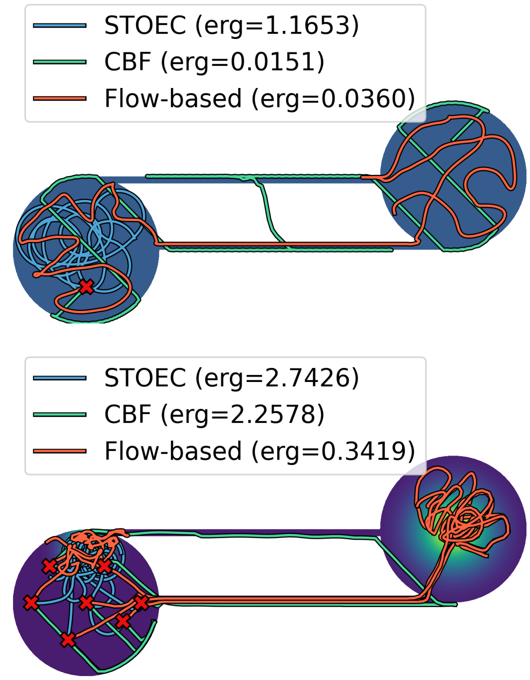

Fig. 2. Results for the rooms map comparing our and prior approaches. From left to right, we have the trajectories generated for the uniform, nonuniform, and multi-agent nonuniform test cases. The ergodic metric value for each trajectory is shown in the figure legend, calculated using the Laplace-Beltrami ergodic metric.

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